GRAVITATIONAL WAVE ANALYSIS WITH PULSAR TIMING ARRAYS

Kristina Islo

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OUTLINE

- Introduction
- Bayesian statistics
- Estimating signal parameters
- Detection significance
- Astrophysical inference

** glossary terms*

GRAVITATIONAL WAVE SPECTRUM

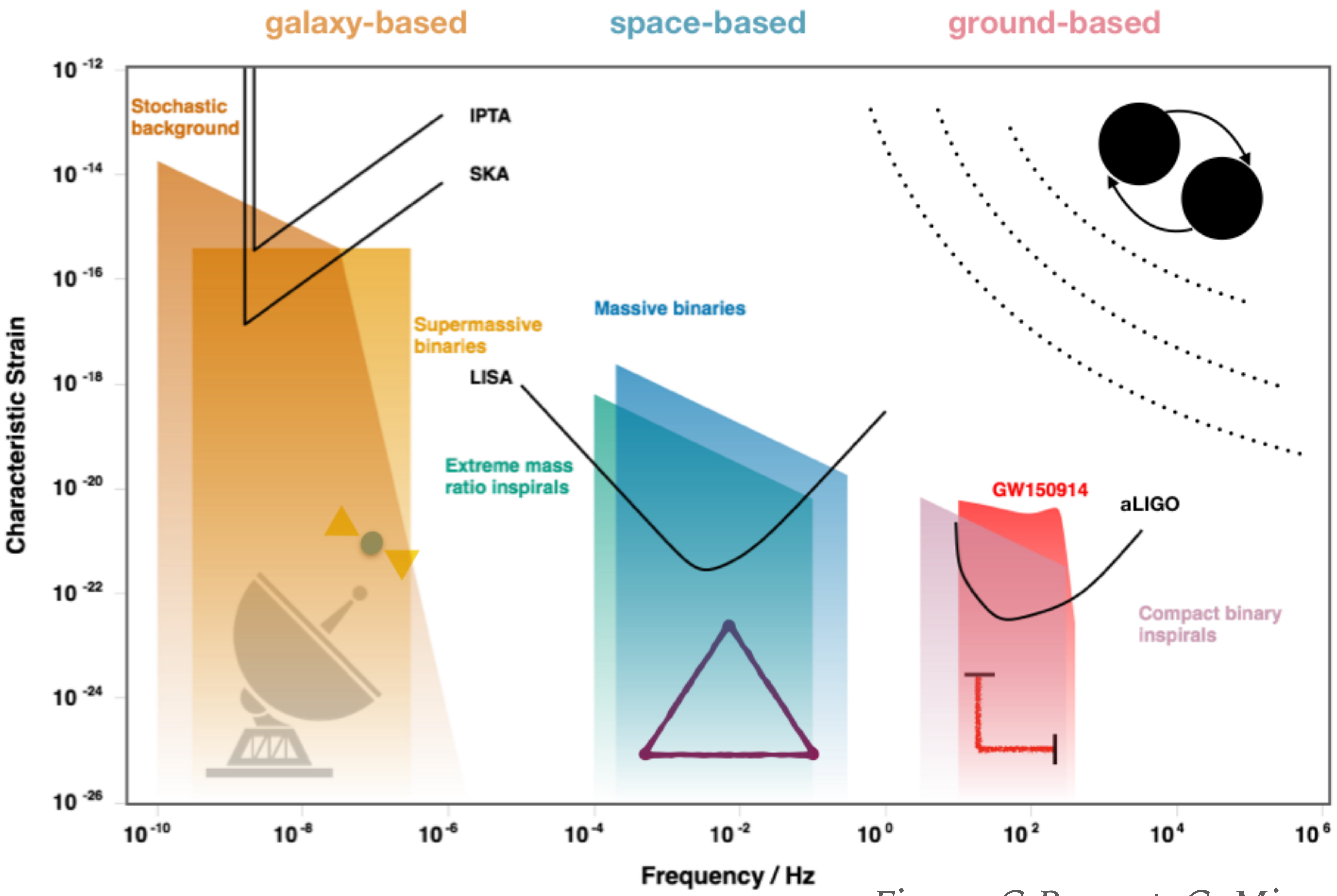
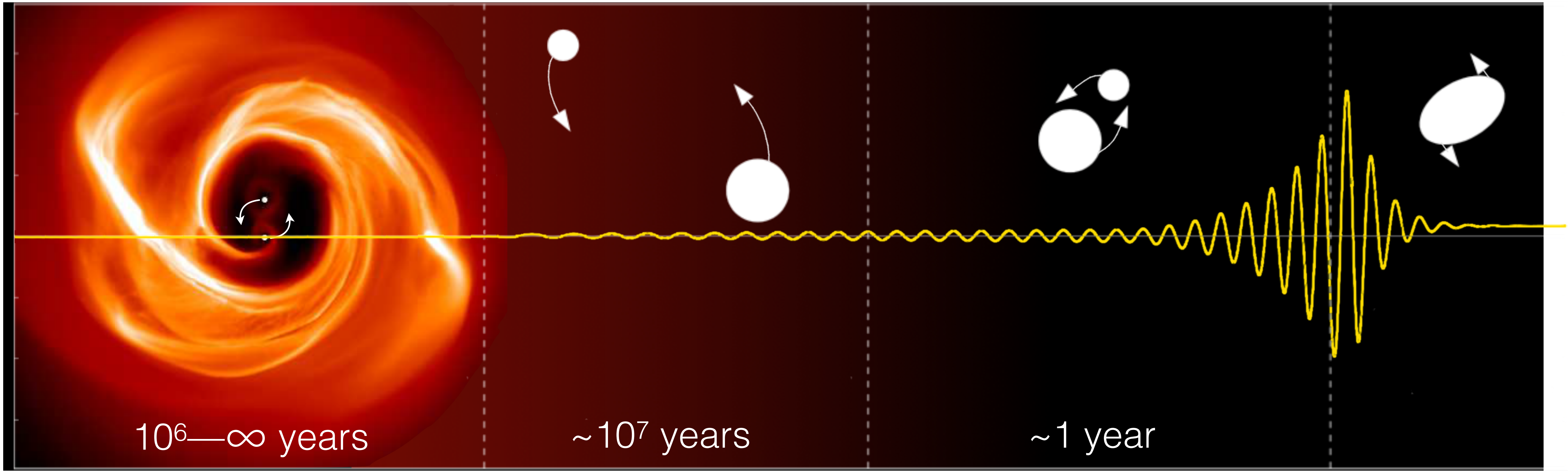


Figure: C.Berry + C. Mingarelli

SUPERMASSIVE BLACK HOLE BINARIES

- Pulsar timing arrays like NANOGrav are sensitive to nanohertz gravitational waves from supermassive black hole binaries
- These binaries are thought to form in the center of merging galaxies

Figure: S. Burke-Spolaor



SIGNATURE OF VARIOUS SIGNALS

- Signals can be classified into two distinct types:
  - **Stochastic** - Described through statistical properties; GW power proportional to variance of signal
  - **Deterministic** - A resolvable waveform we can characterize with typical properties, i.e., amplitude, frequency, phase, etc.

TWO SCHOOLS OF THOUGHT ON HANDLING UNCERTAINTY

Say an astronomer estimates the mass of a neutron star to be

“$M = (1.39 \pm .02)\ M_\odot$ with 90% confidence.”

FREQUENTIST

The long-term frequency of with which you measure the neutron star mass to be in $\{[M - 0.2, M+0.2]\ M_\odot\}$ for any measured mass value M is 90%

BAYESIAN

You are 90% confident the true neutron star mass lies within [1.37 $M_\odot$ - 1.41 $M_\odot$]

FREQUENTIST

- Probability means long-term relative frequency
- You assume measured data is random, but the parameters of the governing hypothesis are fixed but unknown
- Construct a statistic to determine when data are consistent with model
- Probability distribution of statistic
- p-values and confidence intervals

BAYESIAN

- Probability means degree of belief
- The data are fixed, and the parameters of the governing hypothesis are random and mostly unknown
- Prior knowledge is incorporated
- Bayes theorem updates prior in light of additional data
- credible sets and odds ratios

BAYES' THEOREM

$$p(A, B) = p(A)p(B|A) = p(B)p(A|B)$$

"probability that A is true given circumstances B"

"probability that A is true without consideration of B"

"probability that B is true given circumstances A"

$$\boxed{\therefore\ p(B|A) = \frac{p(A|B)p(A)}{p(B)}}$$

USING BAYES' THEOREM: AN EXAMPLE

A test for a disease is 99% accurate. 🔬 $p(\text{positive}|\text{infected}) = 0.99$

1 in 10,000 people have the disease. 📉 $p(\text{infected}) = 0.0001$

What is the probability you get a positive result but aren't infected? 😷 $p(\text{infected}|\text{positive}) = ?$

$$p(\text{infected}|\text{positive}) = \frac{p(\text{positive}|\text{infected})p(\text{infected})}{p(\text{positive})}$$

$$= \frac{0.99 \times 0.0001}{0.99 \times 0.0001 + 0.01 \times (1 - 0.0001)}$$

$$\sim 1\%$$

LIKELIHOOD, PRIOR, POSTERIOR AND EVIDENCE

$\vec{d}$ = *parameters we know well*

$\vec{\theta}$ = *parameters we want to know more about*

$$p(\vec{\theta}|\vec{d}) = \frac{p(\vec{d}|\vec{\theta})p(\vec{\theta})}{p(\vec{d})}$$

Terminology

$p(\vec{\theta}|\vec{d})$: *posterior* probability

$p(\vec{d}|\vec{\theta})$: *likelihood*

$p(\vec{\theta})$: *prior* knowledge

$p(\vec{d})$: *evidence* ← difficult to compute!

MODELING NOISE IN OUR DATA (INCLUDING GWS!)

$$\delta \mathbf{t} = M\epsilon + \mathbf{n}_{\mathrm{white}} + \mathbf{n}_{\mathrm{red}}$$

Timing model

- *spin*
- *spin-down*
- *orbital parameters*
- *dispersion from ISM*

White noise

- *uncorrelated in time*
- *instrumental*
  - *EFAC*
  - *EQUAD*
  - *ECORR*

"Red" noise

- *correlated in time*
- *Primarily astrophysical*
  - *Intrinsic to pulsar*
  - *time-varying DM*
  - *GWs!*

STOCHASTIC BACKGROUND – RED NOISE

- Superposition of gravitational waves from a population of inspiraling supermassive black hole binaries
- Let’s try a Fourier analysis of the background:

$$\mathbf{n}_{\mathrm{red}} = F_{\mathrm{red}}\,\mathbf{a}$$

Fourier coefficients

Fourier basis (sines and cosines)

- We expect largest Fourier coefficients at lower gravitational wave frequencies; we write the red noise power as

$$P_{\mathrm{red}}(f) = A f^{-\gamma}$$

STOCHASTIC BACKGROUND – OBTAINING THE POSTERIOR

Marginalized Likelihood:

$$p(\vec{\theta}, \varphi, \mathbf{a}|\delta t) = p(\delta t|\vec{\theta}, \mathbf{a})p(\mathbf{a}|\varphi)p(\varphi)p(\vec{\theta})$$

Assume multivariate Gaussian priors and integrate over Fourier coefficients:

$$p(\theta, \varphi|\delta t) \propto \frac{\exp\left(-\frac{1}{2}\delta\mathbf{t}^T C^{-1}\delta\mathbf{t}\right)}{\sqrt{\det(2\pi C)}}$$

with Covariance Matrix:

$$C = N + F_{\text{red}}\varphi F_{\text{red}}^T$$

including correlated red noise power with elements for pulsar pairs (a,b) and frequencies (k,l)):

$$\varphi_{(ak),(bl)} = \Gamma_{ab}\rho_k\delta_{kl} + \kappa_{ak}\delta_{ab}\delta_{kl}$$

STOCHASTIC BACKGROUND – RED NOISE

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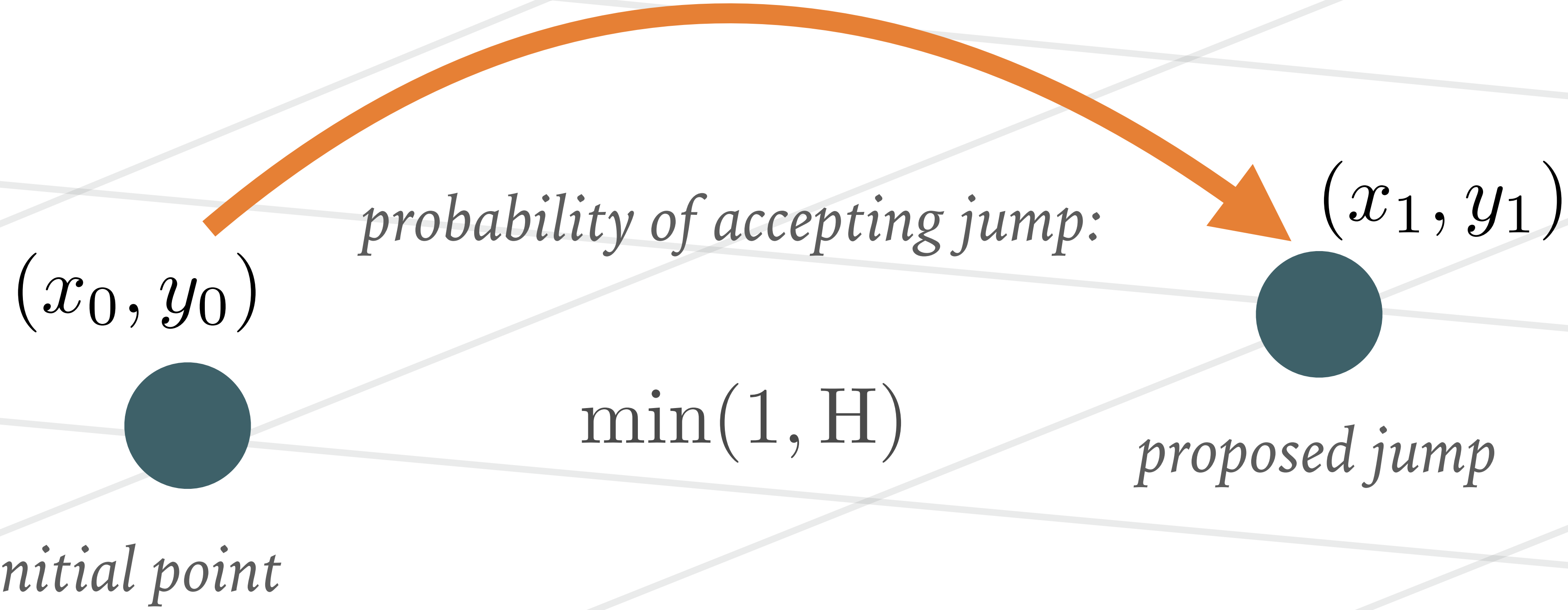
HOW WE SEARCH FOR IT

- Use a Markov-chain Monte-Carlo (MCMC) to efficiently generate samples from the posterior probability distribution that does not require computations of the evidence



probability of accepting jump:

(x_0, y_0)

(x_1, y_1)

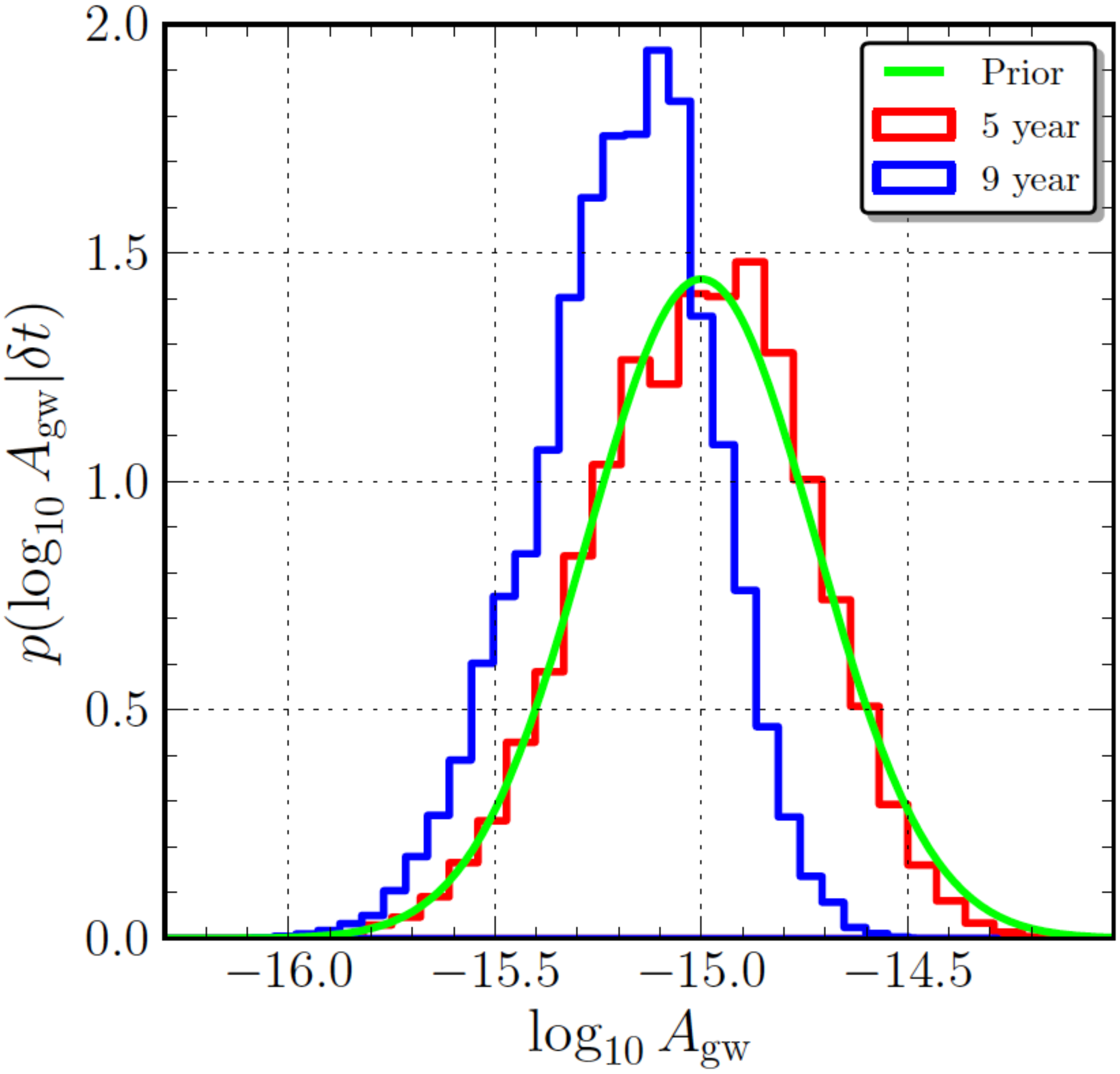
$$\mathrm{min}(1, \mathrm{H})$$

initial point

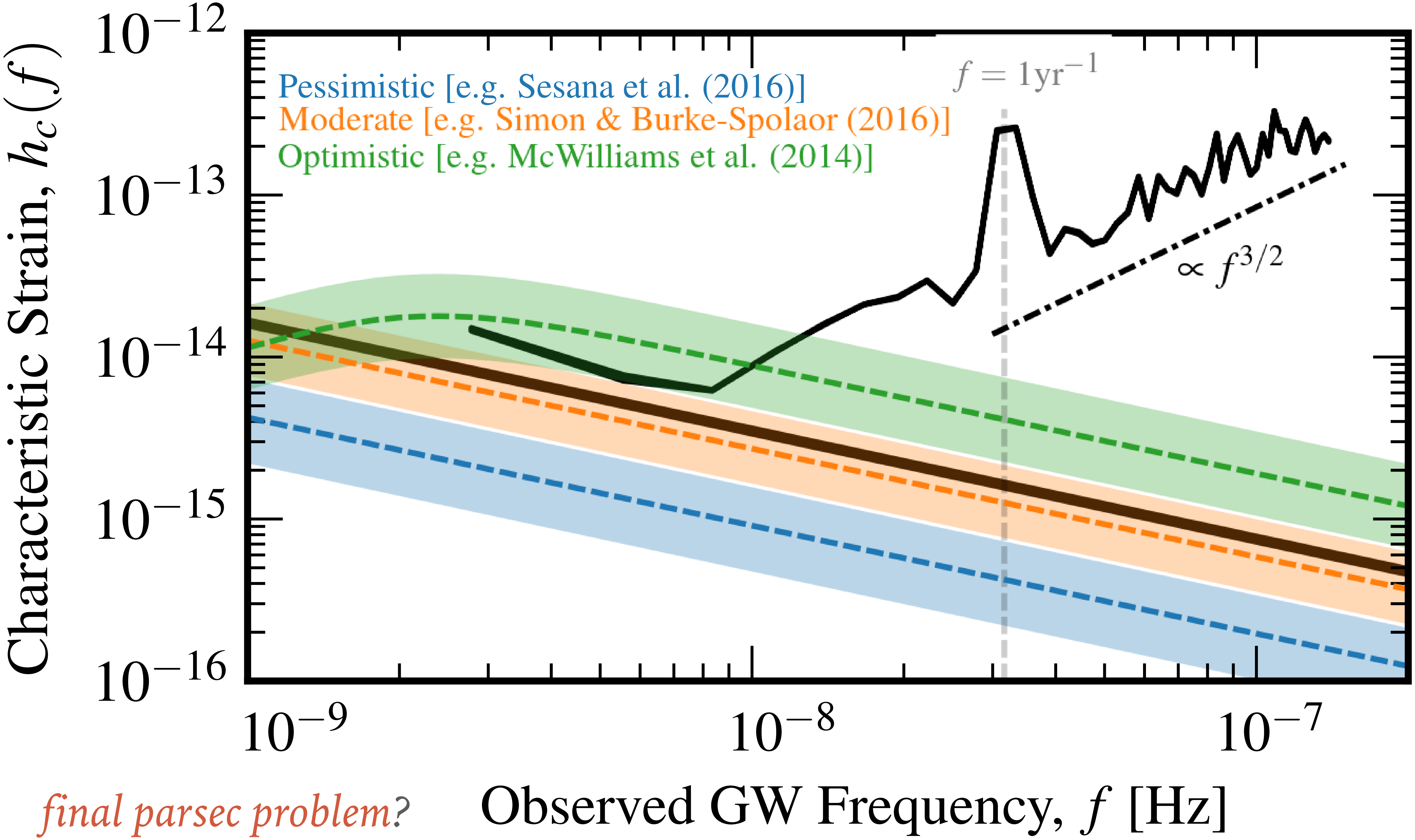
proposed jump

$$H \propto \frac{p(x_1, y_1|\vec{d})}{p(x_0, y_0|\vec{d})}$$

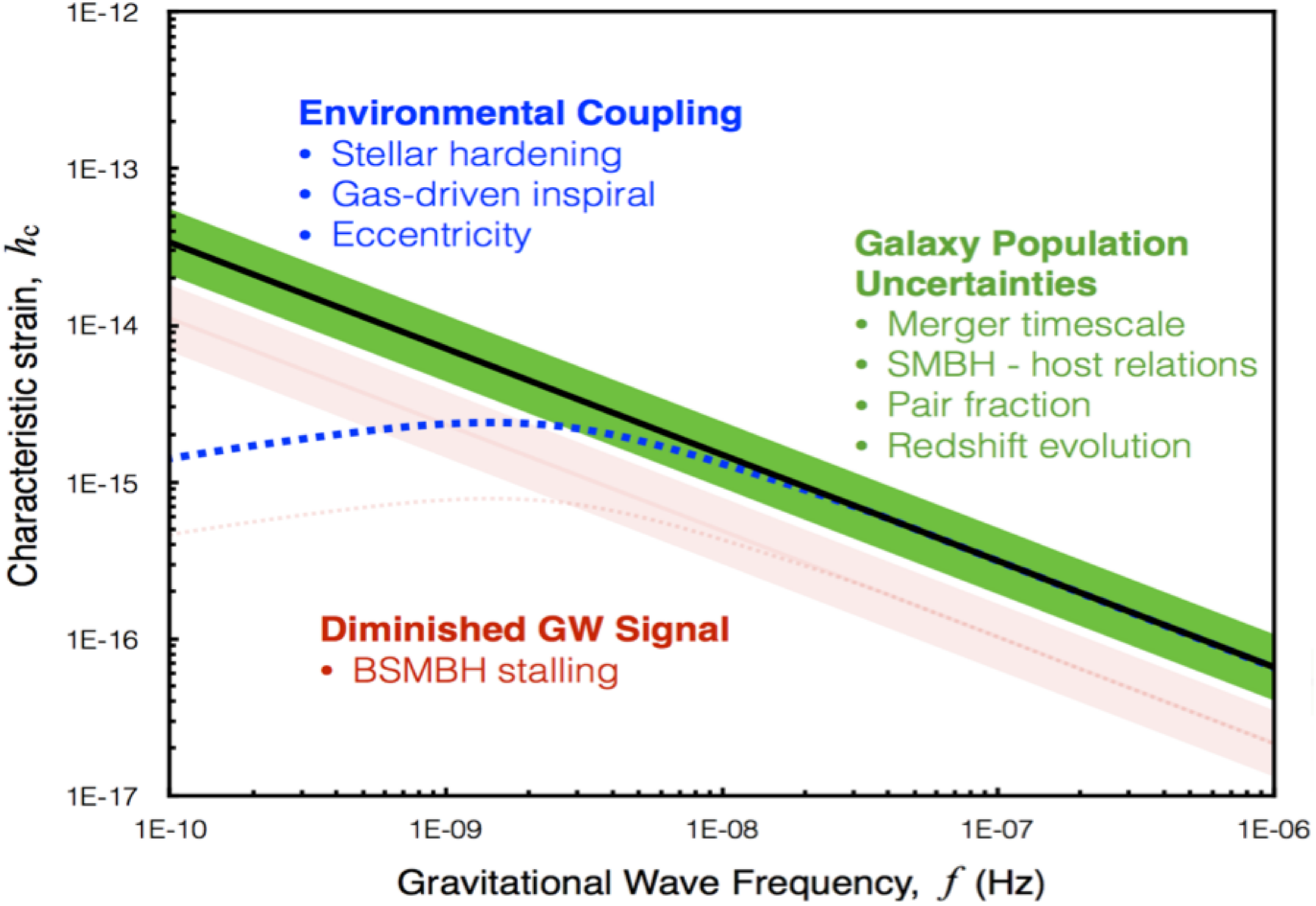
WHAT WE GET - POSTERIOR DISTRIBUTIONS OF SIGNAL PARAMETERS



SENSITIVITY VERSUS MODELS

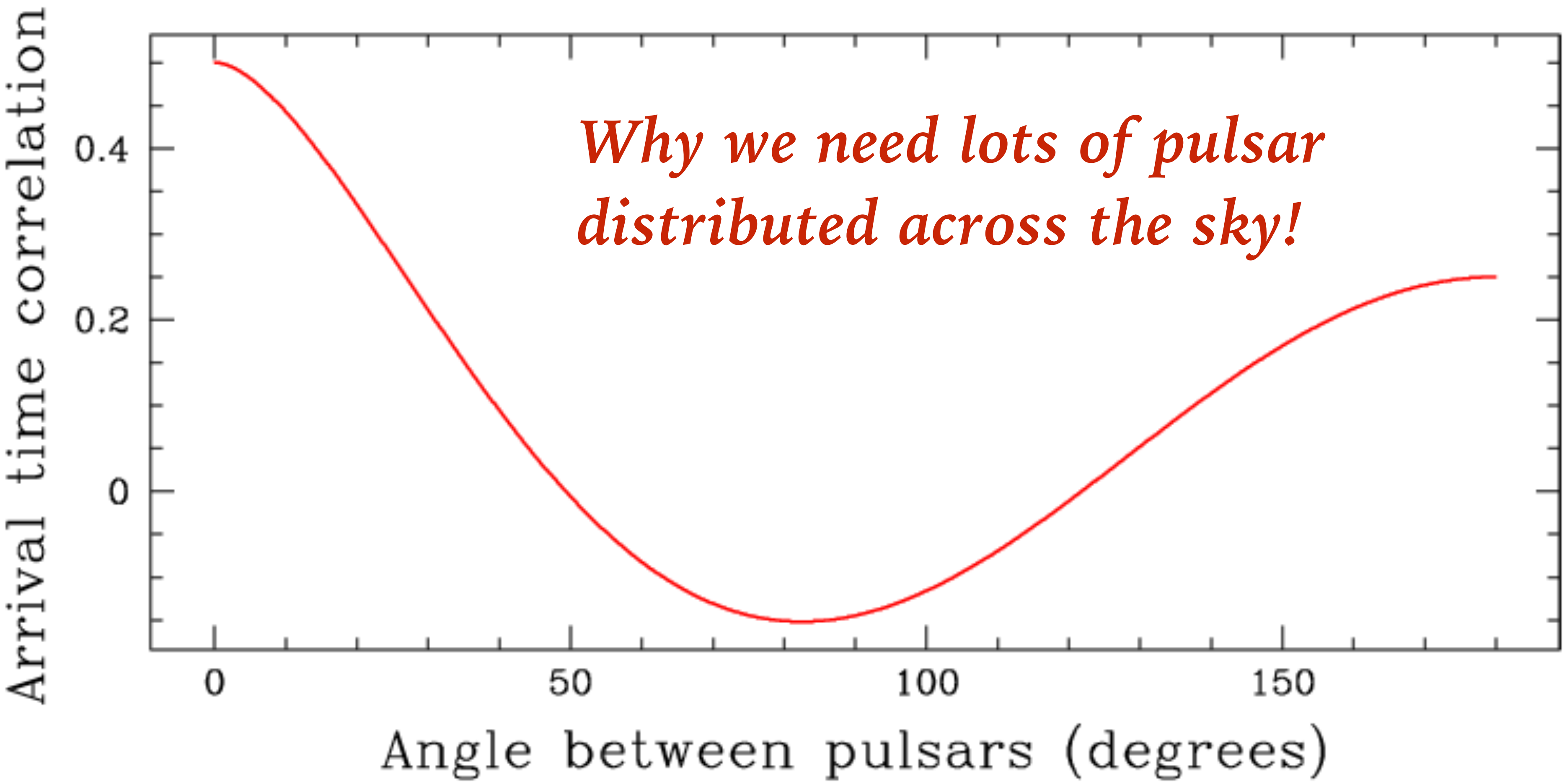


final parsec problem?


Environmental Coupling
• Stellar hardening
• Gas-driven inspiral
• Eccentricity
Galaxy Population Uncertainties
• Merger timescale
• SMBH - host relations
• Pair fraction
• Redshift evolution
Diminished GW Signal
• BSMBH stalling
1E-12
1E-13
1E-14
1E-15
1E-16
1E-17
1E-10
1E-09
1E-08
1E-07
1E-06
Characteristic strain, h_c
Gravitational Wave Frequency, f (Hz)

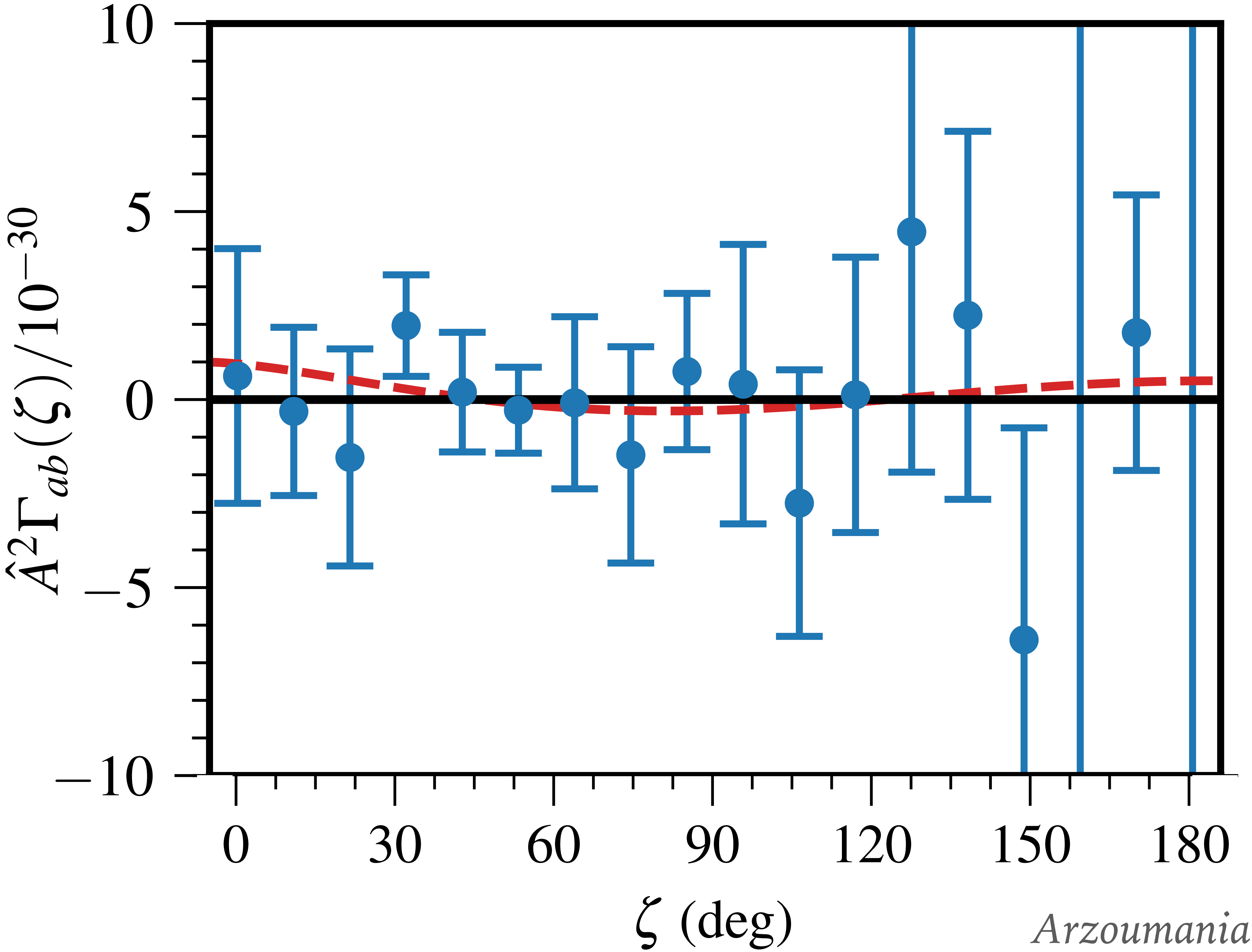
HOW WE SEARCH FOR IT - RED NOISE + Γ

- In addition to searching for the signal variance to get the power of the background, we also look for spatial correlations of the background power across our pulsar array
- Relation between arrival time correlation and angles between pulsar pairs is the Hellings and Downs Curve



Why we need lots of pulsar distributed across the sky!

11-YEAR RESULTS — H&D CORRELATIONS



Arzoumanian+ 2018

BEYOND PARAMETER ESTIMATION

- Detection is a essentially a model selection problem
- Comparing posteriors can give us an odds ratio (e.g. “2:1”)

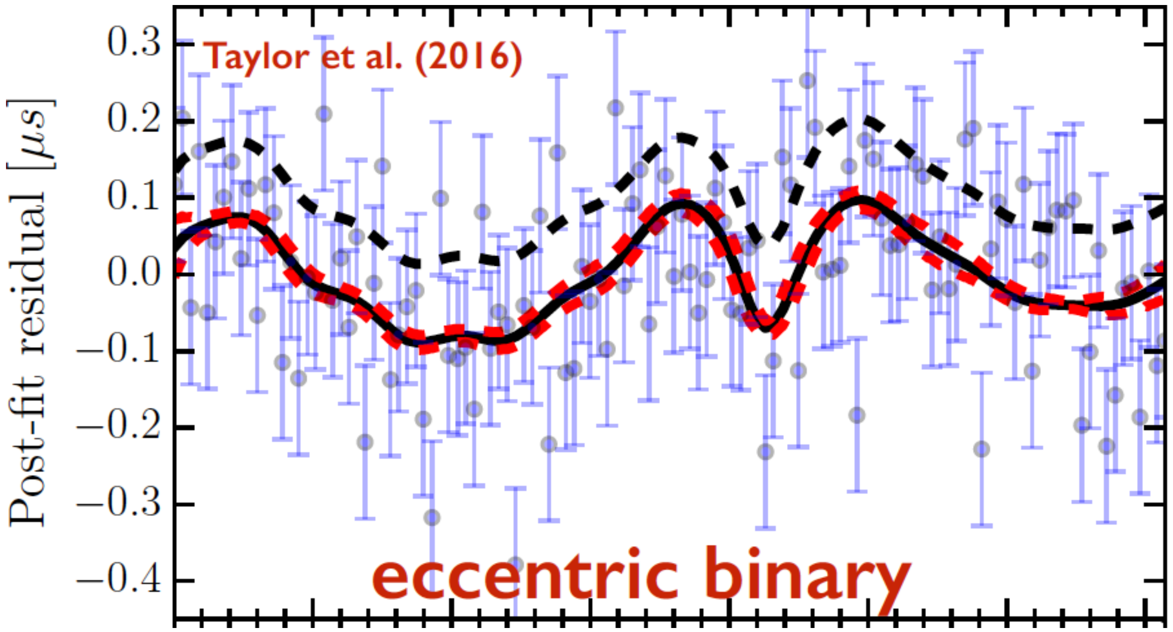
$\mathcal{H}_1 \equiv$ *“red-noise process like a GWB* ***with*** *H&D correlations”*

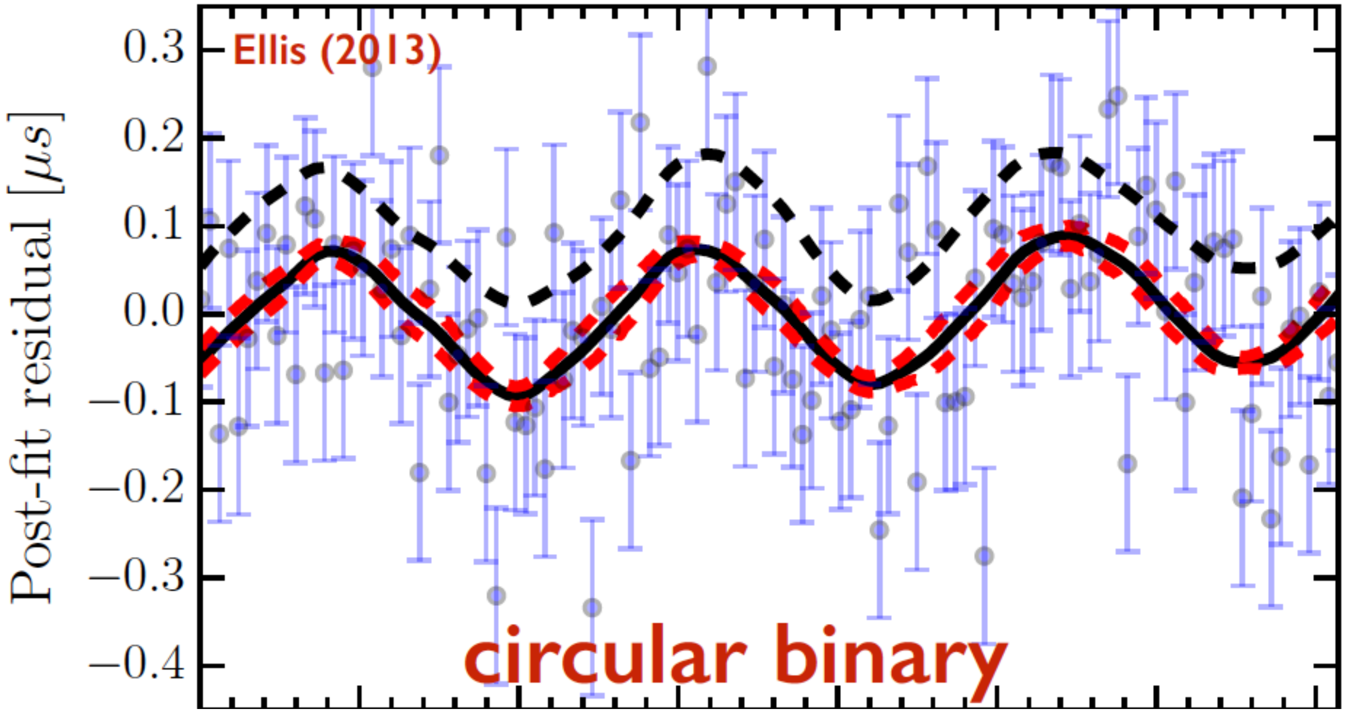
$\mathcal{H}_2 \equiv$ *“red-noise process like a GWB* ***without*** *H&D correlations”*

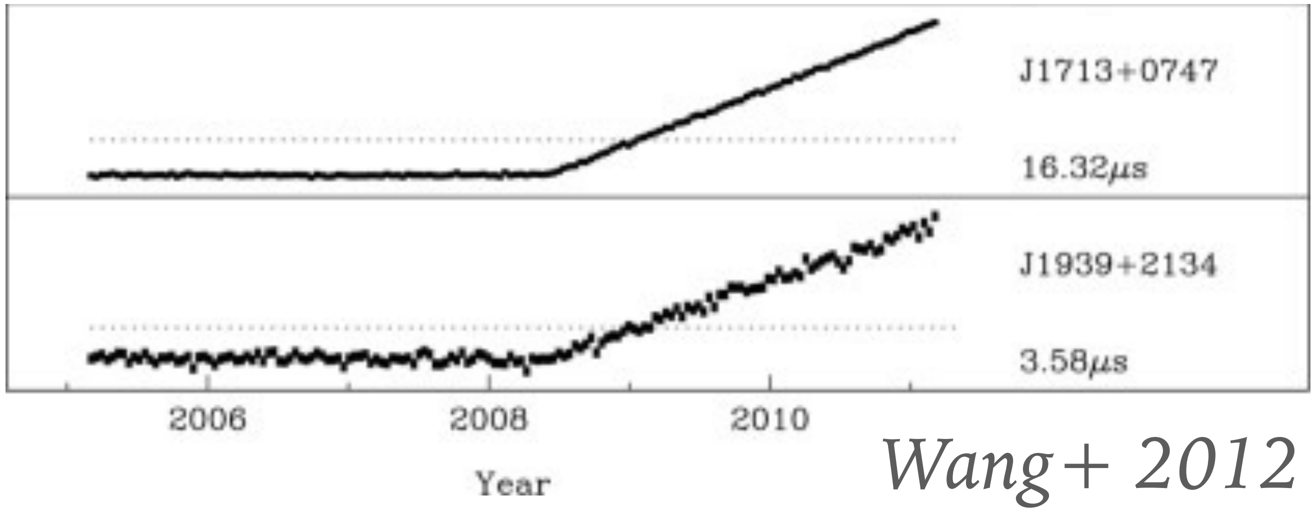
$$\mathcal{P}_{12} = \underbrace{\frac{p(\mathcal{H}_1|\mathbf{d})}{p(\mathcal{H}_2|\mathbf{d})}}_{\textit{posterior odds ratio}} = \underbrace{\frac{p(\mathbf{d}|\mathcal{H}_1)}{p(\mathbf{d}|\mathcal{H}_2)}}_{\textit{Bayes Factor}} \underbrace{\frac{p(\mathcal{H}_1)}{p(\mathcal{H}_2)}}_{\textit{prior odds ratio}}$$

INDIVIDUAL SOURCES

- ➤ Circular Orbit - "continuous waves"
- ➤ Eccentric Orbit
- ➤ Burst with Memory







Wang+ 2012

burst with memory

CONTINUOUS WAVES

- MCMC searches for a GW signal with specific amplitude, frequency and phase

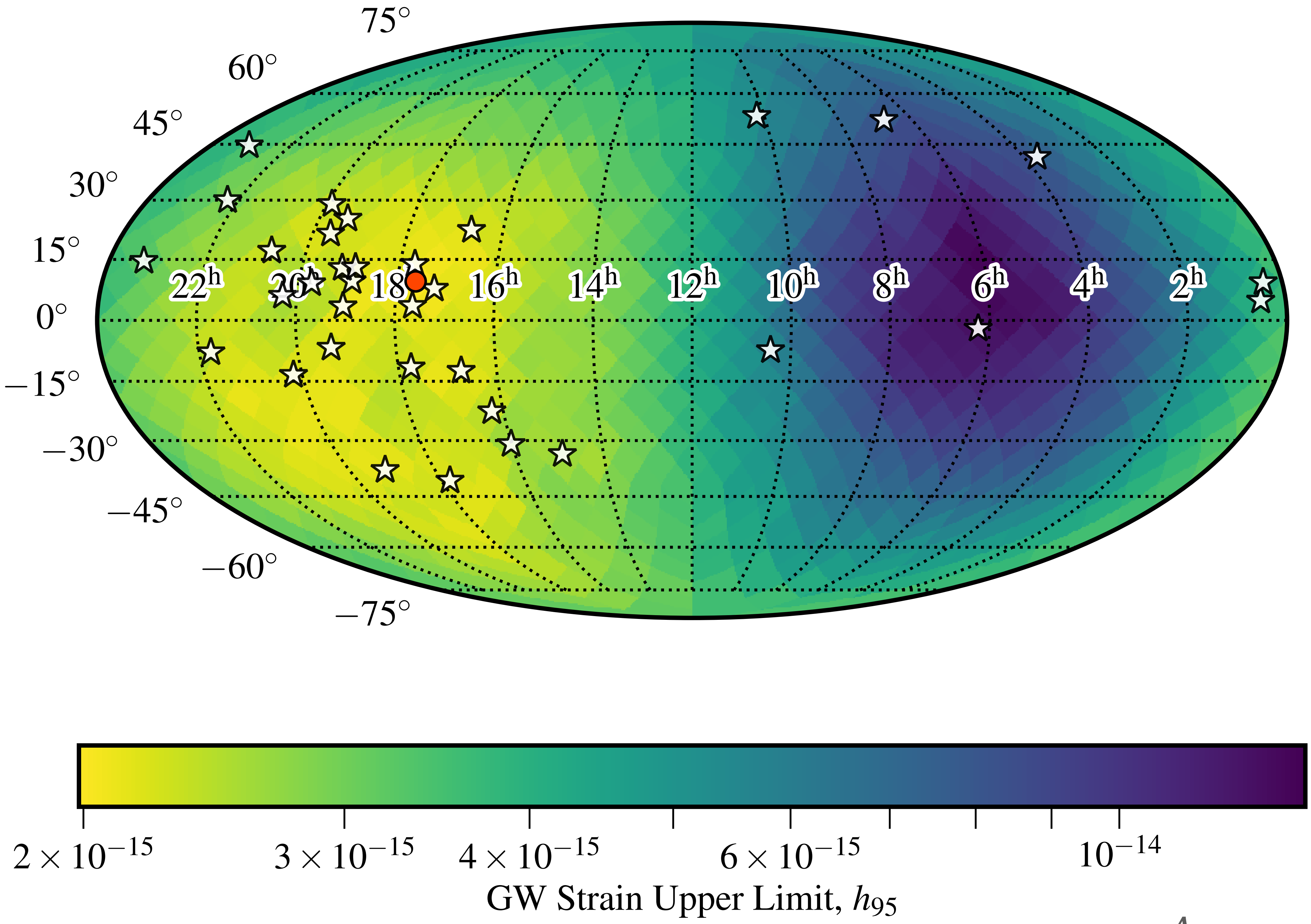
$$h(t) = h_0 \cos(ft + \Psi)$$

- Our model includes parameters:

$$\theta, \phi, \Psi_0, \psi, \mathcal{M}, f, h_0$$

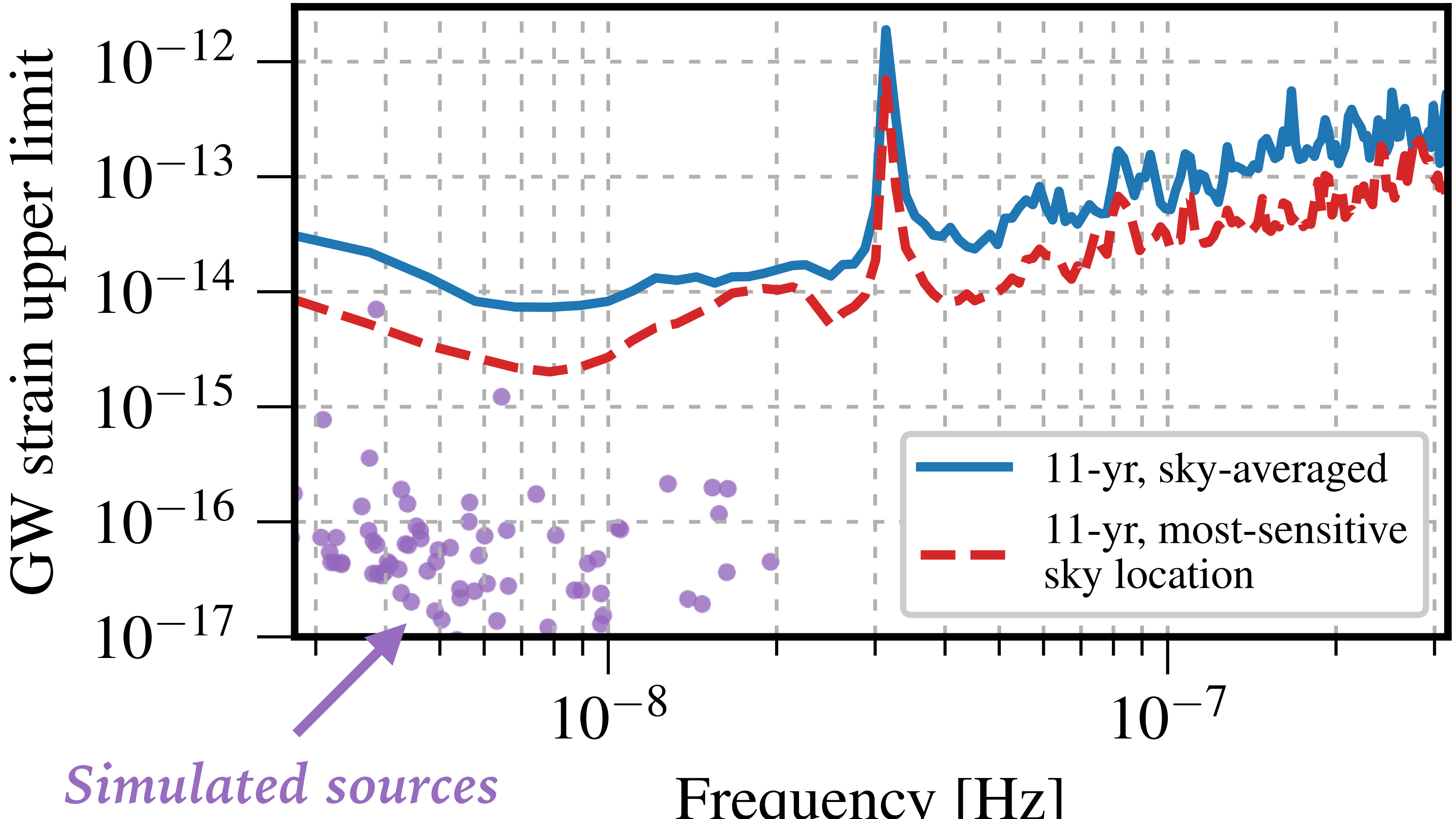
- By searching for specific sources we can place limits that are interesting to astronomers!

ANGULAR SENSITIVITY



Aggarwal+ 2019

SENSITIVITY PER GW FREQUENCY



Simulated sources

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UNIQUE CHALLENGES & OPPORTUNITIES

- We presume the stochastic background is always in our data, hence we cannot “subtract” out the noise to isolate the signal - everything is modeled simultaneously in our MCMCs
- Volume of data necessitates super computing clusters and clever linear algebra to avoid expensive likelihood computations (some combinations of frequentist and bayesian methods)
- Timescales needed for pulsar timing analysis are on the order of other astrophysical phenomena - mistaken identities
    - forays into solar cycles, planetary science, dark matter, etc.

SUMMARY

- PTA data analysis uses Bayesian inference to make strong statements given our data set
- Stochastic and deterministic searches underway to extract information about the presence and character of GW signals from supermassive black hole binaries
- Our limits are ruling out different astrophysical models — constraining our knowledge of how these binaries form and evolve
- pulsar astronomy + general relativity + astrophysics =

FUN!